Quasi-Static Planning Through Contact

Giri Anantharaman Robbie Mones Krishna Suresh

Motivation

When interacting with the world, humans constantly make and break contact. However, methods for enabling robots to perform the same behaviors struggle due to the challenges posed by the dynamics of contact:

- Dynamics non-differentiable at contact events
- Search through contact modes grows combinatorially
- Gradient information about the effects of contact is non-existent before contact occurs

Therefore, recent methods in contact-implicit trajectory optimization solve for control sequences directly without explicitly enumerating changes in dynamics at contact events. We aim to leverage the log-barrier interior-point smoothing method learned in class to solve trajectory optimization problems which can handle making and breaking contact in a quasi-static environment.

Method

We use quasi-static dynamics to simplify the problem and use the log-domain trick as described in class to ensure the complementarity problem is solved by construction. This allows us to start with a large ρ to enable force-at-a-distance and non-physicality to allow the solver to find a solution in the right region. Then we can decrease ρ progressively to ensure contact forces occur only at contact. Below is a visualization of the perceived force at various points in the configuration space for different values of ρ.

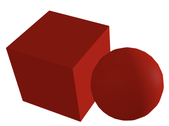

Figure 1. Block ball system for visualizations of contact force bellow.

For large values of ρ, configurations far away from contact still experience forces, allowing for the optimizer to capture the gross motions of the objects. Then as ρ is reduced the contact forces are only applied at the real contact locations ensuring the realism of the dynamics.

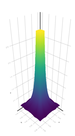

Figure 2. $\rho = 2$

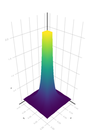

Figure 3. $\rho = 1$

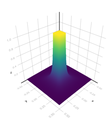

Figure 4. $\rho = 0.1$

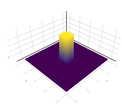

Figure 5. $\rho = 0.001$

Quasi static Contact Dynamics and Optimization

In this section we describe the dynamics of the simple 2D system, that includes a T shaped box and a circle. Circle is actuated, meaning we are able to control the circle to push and rotate the box.

Equations of Motion

Mass matrix (circle is assumed to be point mass, with no inertia) and spring constant matrix (we model the interaction between circle and box like a spring damper. State (position of circle and box only, velocities are assumed to be zero in quasi static formulation) and Controls (2 dimensional since only circle can be actuated):

$$M = \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & m & \\ & & & m \end{bmatrix} K = \begin{bmatrix} k & & & \\ & k & & \\ & & 0 & \\ & & & 0 \end{bmatrix} B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{q} = \begin{bmatrix} q_x^c \\ q_y^c \\ q_x^b \\ q_y^b \end{bmatrix} \mathbf{u} = \begin{bmatrix} \Delta q_x^c \\ \Delta q_y^c \end{bmatrix} = \begin{bmatrix} q_d - q_x^c \\ q_d - q_y^c \end{bmatrix}$$

In addition to these we also define, the following: Signed distance function: $\phi(q)$ Delta q: δq Contact Jacobian: J and Contact force applied by the box: λ. Putting it all together:

Equation of motion:

$$M\ddot{q} + P(q) = KBu + J^T\lambda$$

With these basic ingredients, we now formulate the quasi static contact optimization problem as follows:

$$min_{\delta q} \frac{1}{2h}\delta q^T M \delta q + \frac{h}{2}\delta q^T KB\delta q + h(P(q) - KBu)\delta q$$
$$\text{subject to } \phi(q_k + \delta q) \geq 0$$

We linearize the signed distance function constraint, introduce a slack variable, and formulate the log barrier version of the same problem as:

$$min_{\delta q, s} C(\delta q) - \rho \log(s)$$
$$\text{subject to } \phi(q_k) + hJ\delta q - s = 0$$
$$s \geq 0$$

*Change of Variables

$$s = \sqrt{\rho}e^{\sigma}$$
$$\lambda = \sqrt{\rho}e^{-\sigma}$$

*Lagrangian

$$\mathcal{L}(\delta q, s, \lambda) = C(\delta q) - \rho \log(s) - \lambda^T(\phi(u) + J\delta q - s)$$

$$\nabla_{\delta q}\mathcal{L} : \frac{M\delta q}{h} + hKB\delta q + hP(q) - KBuh - J^T\sqrt{\rho}e^{-\sigma}h$$

$$\nabla_s \mathcal{L} : s\lambda = \rho$$

$$\nabla_\lambda \mathcal{L} : -\phi(q_k) - J\delta q h + \sqrt{\rho}e^{\sigma}$$

Preliminary Results

The following figures show some preliminary results of a translation-only two-dimensional push-T task. These show good performance after five optimization rounds, first with $\rho = 1$ for 500 iterations, then with $\rho = 10^{-1}$, 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5} for 1000 iterations.

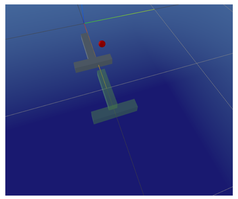

Figure 6. The initial push-T configuration. The object needing to be pushed is gray, the target position of the object is green, the actuator is red.

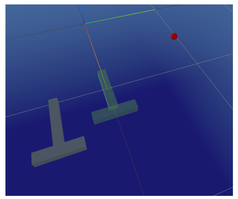

Figure 8. The final push-T configuration after one round of optimization with 500 iterations with $\rho = 1$.

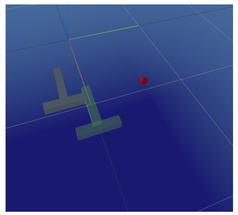

Figure 10. The final push-T configuration after a second round of optimization with 1000 iterations with $\rho = 10^{-1}$.

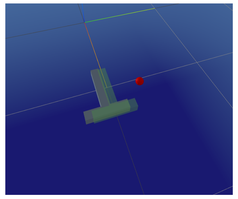

Figure 7. The final push-T configuration after a third round of optimization with 1000 iterations with $\rho = 10^{-2}$.

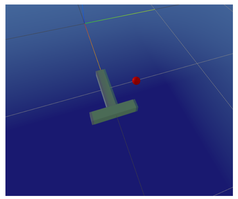

Figure 9. The final push-T configuration after a fourth round of optimization with 1000 iterations with $\rho = 10^{-3}$.

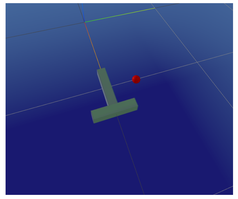

Figure 11. The final push-T configuration after a fifth round of optimization with 1000 iterations with $\rho = 10^{-4}$.

The following figures show some preliminary results of a translation-only two-dimensional push-T task. These show good performance after just two optimization rounds, first with $\rho = 10^{-3}$ for 500 iterations, then with $\rho = 10^{-4}$ for 1000 iterations, skipping early optimization rounds with larger ρ.

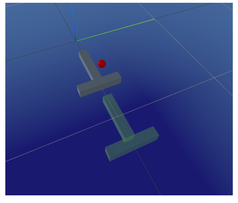

Figure 12. The initial push-T configuration. The object needing to be pushed is gray, the target position of the object is green, the actuator is red.

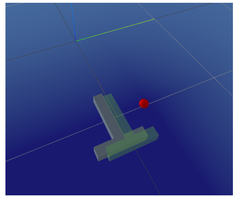

Figure 13. The final push-T configuration after one round of optimization with 500 iterations with $\rho = 10^{-3}$.

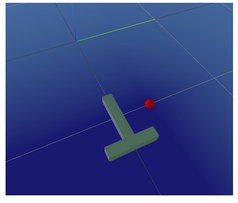

Figure 14. The final push-T configuration after a second round of optimization with 1000 iterations with $\rho = 10^{-4}$.